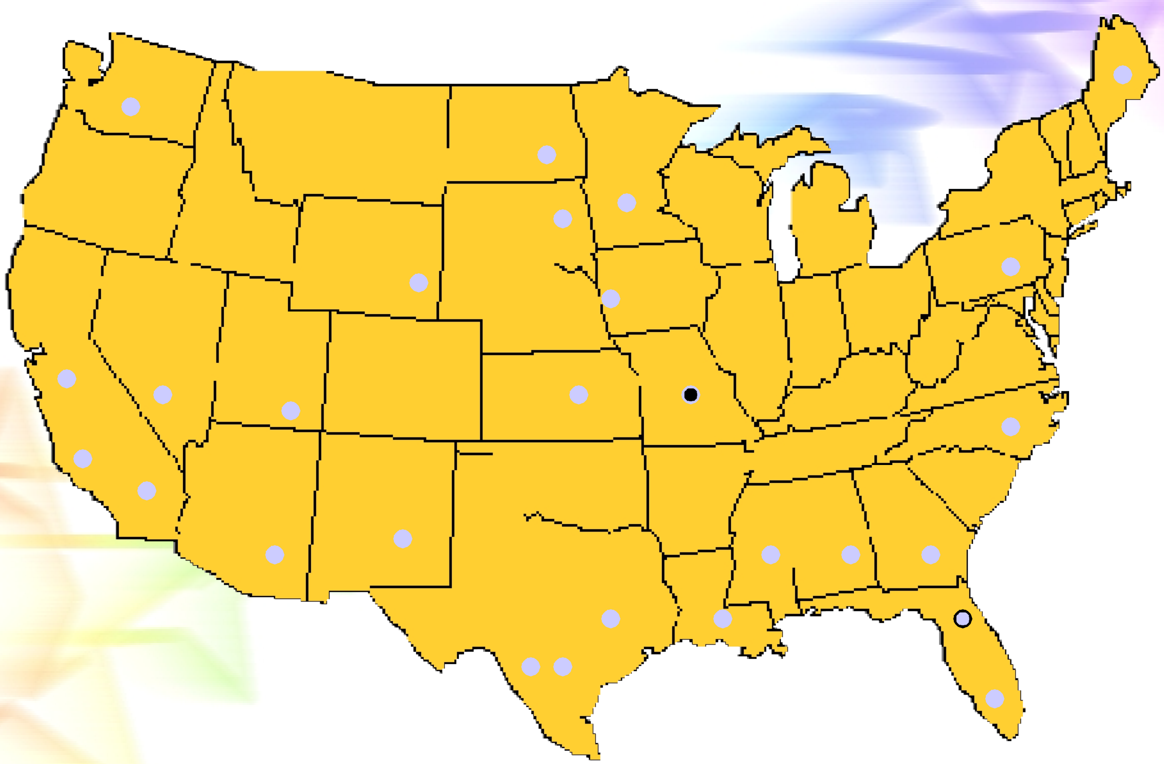
Hard Problems

- Some problems are hard to solve.
  - No polynomial time algorithm is known
  - Most combinatorial optimization problems are hard
  - Popular NP-hard problems:
    - Traveling Salesman
    - N-Queens
    - Bin packing
    - 0/1 knapsack
    - Graph partitioning
    - and many more ….

Traveling Salesperson Problem (TSP)

- Let G be a weighted directed graph.
- A tour in G is a cycle that includes every vertex of the graph.
- TSP => Find a tour of shortest length.
- Problem is NP-hard.

Applications Of TSP



- Home city
- Visit city

Applications Of TSP

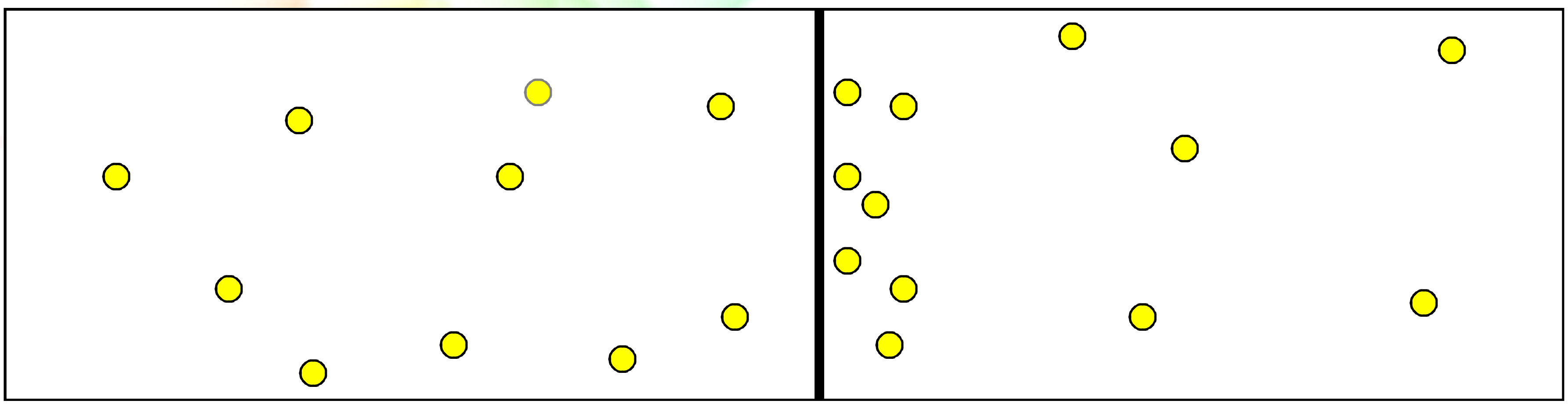
- Each vertex represents a city that is in Joe's sales district.
- The weight on edge (u,v) is the time it takes Joe to travel from city u to city v.
- Once a month, Joe leaves his home city, visits all cities in his district, and returns home.
- The total time he spends on this tour of his district is the travel time plus the time spent at the cities.
- To minimize total time, Joe must use a shortest-length tour.

Applications Of TSP

- Tennis practice.
- Start with a basket of approximately 200 tennis balls.
- When balls are depleted, we have 200 balls lying on and around the court.
- The balls are to be picked up by a robot (more realistically, the tennis player).
- The robot starts from its station visits each ball exactly once (i.e., picks up each ball) and returns to its station.

Applications Of TSP

Robot Station

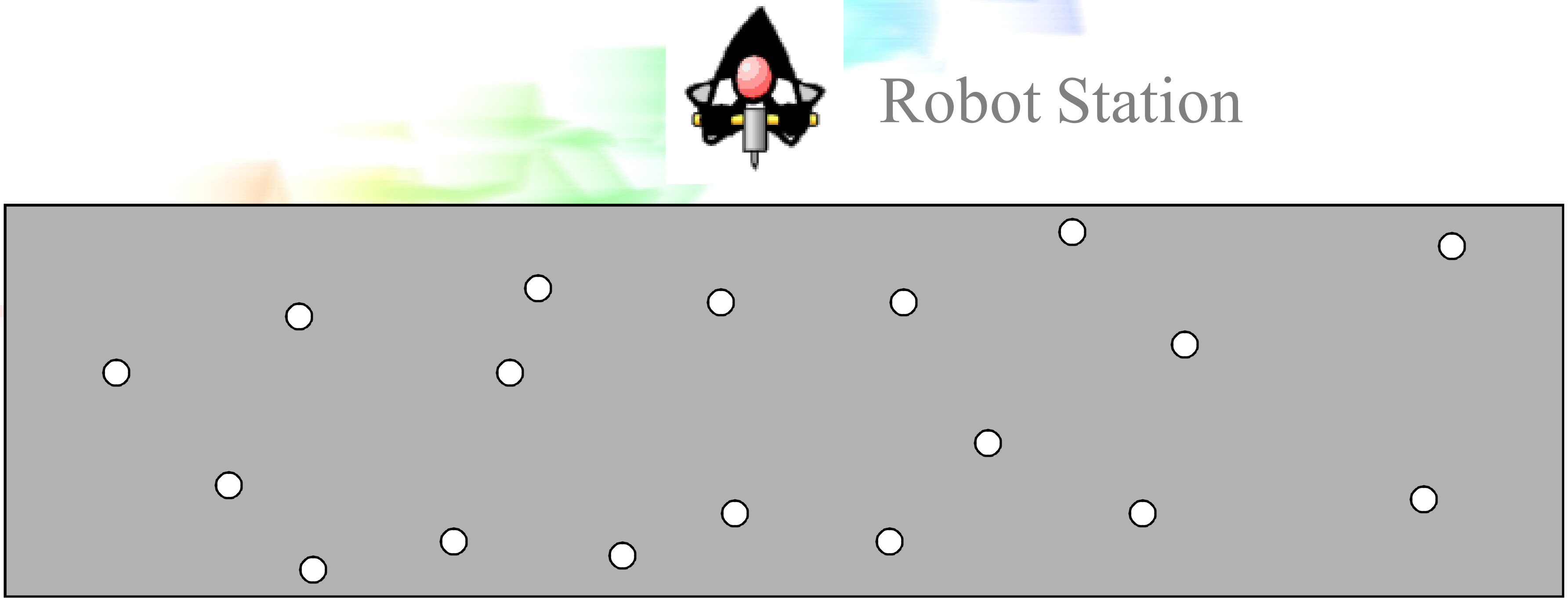


Applications Of TSP

- 201 vertex TSP.
- 200 tennis balls and robot station are the vertices.
- Complete directed graph.
- Length of an edge (u,v) is the distance between the two objects represented by vertices u and v.
- Shortest-length tour minimizes ball pick up time.
- Actually, we may want to minimize the sum of the time needed to compute a tour and the time spent picking up balls using the computed tour.

Applications Of TSP

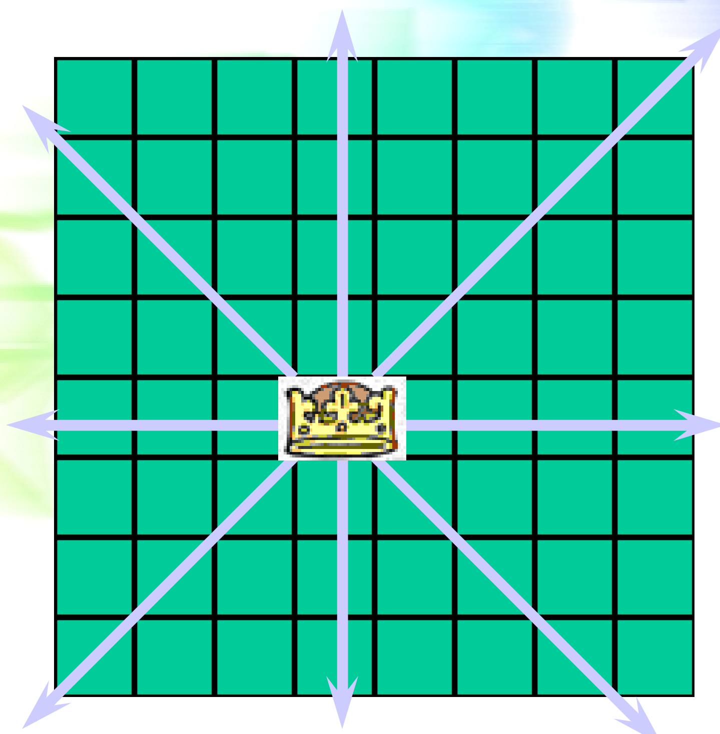
- Manufacturing.
- A robot arm is used to drill n holes in a metal sheet.



n+1 vertex TSP.

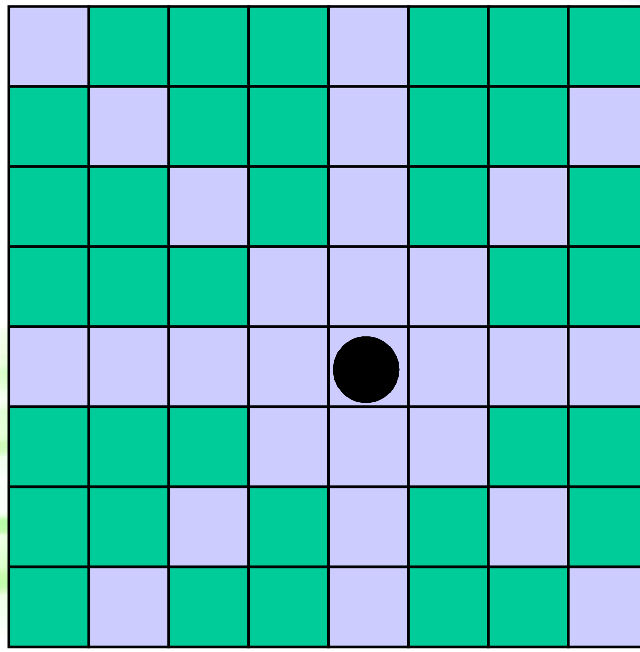
n-Queens Problem

A queen that is placed on an n x n chessboard, may attack any piece placed in the same column, row, or diagonal.



8x8 Chessboard

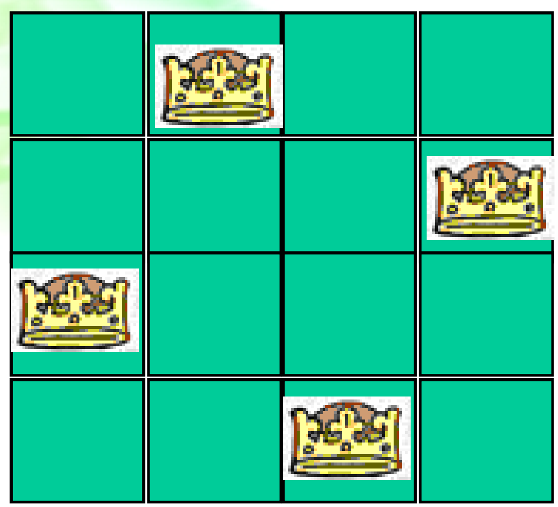
8 Queens Problem



Place 8 queens on a 8x8 chessboard in such a way that the queens cannot check each other.

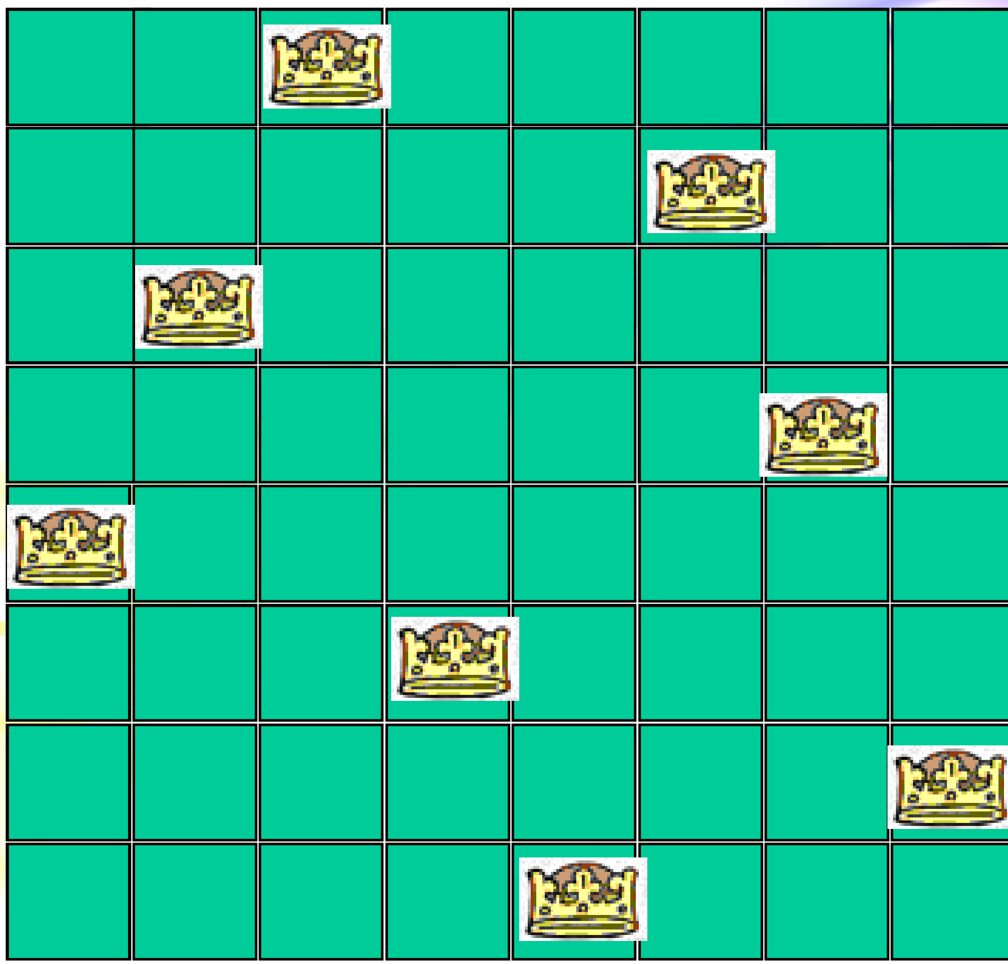
4-Queens Problem

Can 4 queens be placed on an 4 x 4 chessboard so that no queen may attack another queen?



4x4

One possible solution for 8-Queens Problem



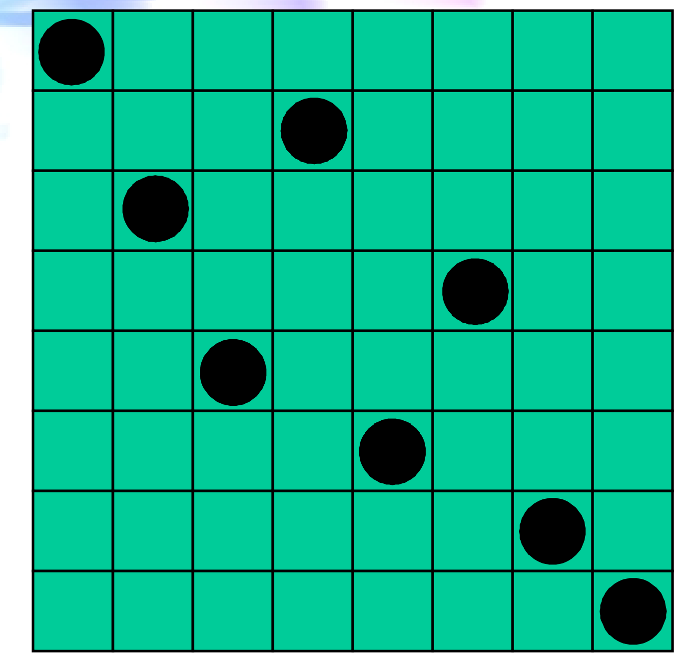
8x8

8 Queens - Representation

Genotype: a permutation of the numbers 1 through 8

1	3	5	2	6	4	7	8

↔



Phenotype: a configuration

Difficult Problems

- Many require you to find either a subset or permutation that satisfies some constraints and (possibly also) optimizes some objective function.
- May be solved by organizing the solution space into a tree and systematically searching this tree for the answer.

Permutation Problems

- Solution requires you to find a permutation of n elements.
- The permutation must satisfy some constraints and possibly optimize some objective function.
- Examples.
  - TSP.
  - n-queens.
    - Each queen must be placed in a different row and different column.
    - Let queen i be the queen that is going to be placed in row i.
    - Let c_i be the column in which queen i is placed.
    - $c_1, c_2, c_3, \ldots, c_n$ is a permutation of [1,2,3, …, n] such that no two queens attack.

Solution Space

- Permutation problem.

  n = 2, {12, 21}

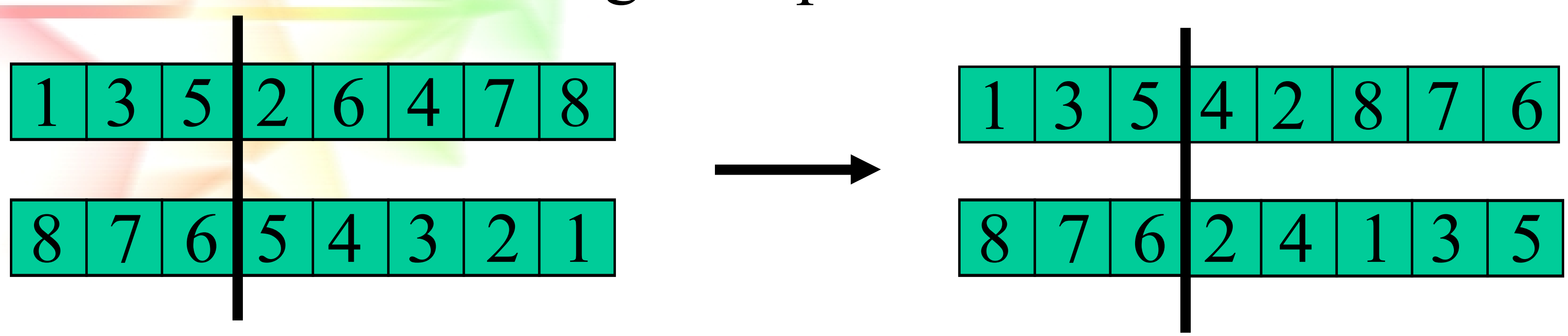
  n = 3, {123, 132, 213, 231, 312, 321}

- Solution space for a permutation problem has n! members.
- Nonsystematic search of the space for the answer takes O(pn!) time, where p is the time needed to evaluate a member of the solution space.

8 Queens - Operators

Mutation: exchanging two numbers

Crossover: combining two parents



8 Queens - Fitness & Selection

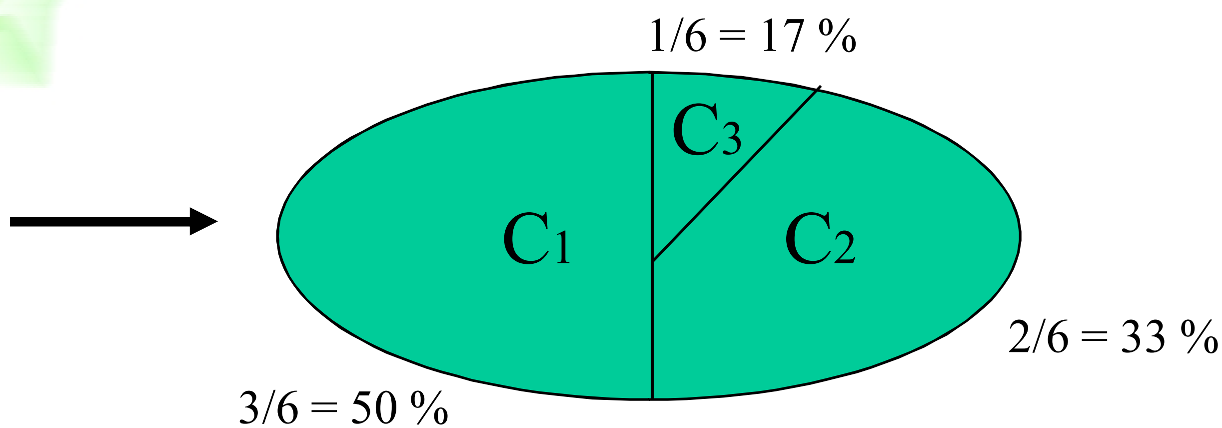
Fitness: penalty of one queen is equal to the number of queens she can check.
The fitness of the configuration is equals the sum of the penalties of all queens.

Selection: using a roulette wheel

fitness(C_1) = 1

fitness(C_2) = 2

fitness(C_3) = 3



Assignment

Q.1)Write a short note on Simplified NP hard problem.
Q.2)Write a note on NP hard graph problem.
Q.3)Writ a note on NP Hard scheduling problem